Exercise 45

Use logarithmic differentiation to find the derivative of the function.

$$y = x^{\sin x}$$

Solution

Take the natural logarithm of both sides and use the properties of logarithms to simplify the right side.

$$\begin{aligned}
\ln y &= \ln x^{\sin x} \\
&= \sin x \ln x
\end{aligned}$$

Differentiate both sides with respect to x.

$$\begin{aligned}
\frac{d}{dx}(\ln y) &= \frac{d}{dx}(\sin x \ln x) \\
\frac{1}{y} \cdot \frac{d}{dx}(y) &= \left[\frac{d}{dx}(\sin x)\right] \ln x + \sin x \left[\frac{d}{dx}(\ln x)\right] \\
\frac{1}{y} \cdot \frac{dy}{dx} &= (\cos x) \ln x + \sin x \left(\frac{1}{x}\right) \\
\frac{1}{y}\frac{dy}{dx} &= \cos x \ln x + \frac{\sin x}{x} \\
\frac{dy}{dx} &= y\left(\frac{x \cos x \ln x + \sin x}{x}\right) \\
&= x^{\sin x}\left(\frac{x \cos x \ln x + \sin x}{x}\right) \\
&= \frac{x \cos x \ln x + \sin x}{x^{1-\sin x}}
\end{aligned}$$